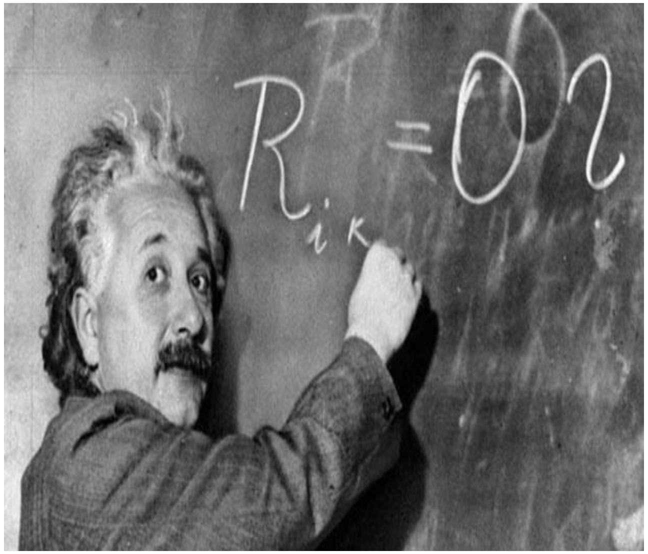
Brief History of Albert Einstein's



Albert Einstein

Born	:	14 March 1879
Place	:	Ulm
Died	:	18 April 1955 at Princeton
Nationality	:	Germany

CARRIER LIFE:

In 1901, the year he gained his diploma, he acquired Swiss citizenship and, as he was unable to find a teaching post, he accepted a position as technica assistant in the Swiss Patent Office.

In 1905 he obtained his doctor's degree.

In 1909 he became Professor Extraordinary at Zurich,

In 1911 Professor of Theoretical Physics at Prague.

Einstein's Famous equation: $E = mc^2$

Einstein's work led to some startling results, which today still seem counterintuitive at first glance even though his physics is usually introduced at the high school level.

One of the most famous equations in mathematics comes from special relativity. The equation $E = mc^2$— means "energy equals mass times the speed of light squared." It shows that energy (E) and mass (m) are interchangeable; they are different forms of the same thing. If mass is some how totally converted into energy, it also shows how much energy would reside inside that mass: quite a lot. (This equation is one of the demonstrations for why an atomic bomb is so powerful, once its mass is converted to an explosion.

Einstein's brain was different from other people's

Einstein's brain actually *looks* different from yours. This is an actual photo of Einstein's brain, which was preserved in formalin by pathologist Thomas Harvey after Einstein's death in 1955. A new study of this photo and others of Einstein's brain reveal an unusually complex pattern of convolutions in the prefrontal cortex, which is important for abstract thinking.

How did Einstein's brain come to undergo so much scrutiny? Pathologist Thomas Harvey performed an autopsy on Einstein shortly after his death in 1955. At that time, he removed Einstein's brain and preserved it in formalin. He took dozens of black-and-white photos of the brain. Later, he cut Einstein's brain up into 240 blocks, took tissue samples from each block, mounted them onto microscope slides and distributed the slides to some of the world's best neuropathologists.

CHAPTER-I

Review of Newtonians Mechanics

Definition

I)Space: Absolute space it to own nature without relation to anything external remains always similar and immovable.

ii)Time: Absolute true and mathematical time of itself and from it is own nature flows equably without relation to an thing external

Newton law of motion:

Newton First law:

A body continuous in it's state of rest or uniform motion unless it is acted upon by external forces

Newton Second law

The rate of change of is proportional to impressed force in direction of force itself

$$(F = \frac{d}{dt}(mv))$$

Newton third law

To every action there is equal and opposite reaction.

Inertial system

The reference system where newton first low of motion is valid is called as inertial system.

Q.1) Show that particle moves in a straight lines with constant velocity.

Solution: From Newton second low of motion

We have equation of motion given by

$$\bar{F} = \frac{d}{dt}(m\bar{v}).............(1)$$

Where $\bar{F}$ = force acting on particle having intertie mass m moving with velocity $\bar{V}$
Since inertial mass m is assume to be constant.

$$\bar{F} = m\frac{d\bar{v}}{dt}m\bar{a}........(2)$$

Where $\bar{a} = \frac{d\bar{v}}{dt}$ is acceleration of particle.

If in certain inertial frame $\bar{F} = 0$, then $\bar{a} = 0$
Let $\bar{r}$ be the position vector of a moving particle at time.

$$\bar{V} = \frac{d\bar{r}}{dt}$$

$$\Rightarrow \frac{d\bar{v}}{dt} = \frac{d^2\bar{r}}{dt^2}$$

$$\bar{a} = \frac{d^2r^2}{dt^2}..........(3)$$

Since in an inertial frame, the body is not under the influence of any forces

$$\Rightarrow \bar{F} = 0 \Rightarrow m\bar{a} = 0$$

From (2) $m \neq 0 \Rightarrow \bar{a} = 0$

$$\frac{d^2\bar{r}}{dt^2} = 0$$

Integrating we get

$$\frac{d\bar{r}}{dt} = \bar{A}.............(4)$$

$$\Rightarrow \bar{V} = \vec{A}$$

Where $\bar{A}$ be a constant vector of integration and hence velocity is constant

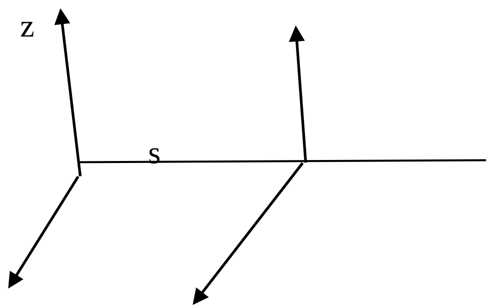
$$\bar{r} = \bar{A}t + \bar{B}..............(5)$$

Where $\vec{B}$ is a constant of integration
The equations (5) represent a straight line.Thus in inertial frame body mass in straight line with constant velocity.

Q2.Obtain Galilean Transformation equation for two initial frame in relative motion.

Solution : Consider any two inertial Frames S and S'. Let frame S' is moving with velocity $\bar{v}$ w.r.to frame S such that their origin is coincide with t=0 and t'=0

Let $P(\bar{r},t)$ and $(\bar{r},t')$ be the point associate with event in the frame S and S' respectively where $\bar{r}=(x,y,z)$ $\bar{r}'=(x',y',z')$ as shown in figure



By triangle law of vector addition ,we get

$$\bar{r}'=\bar{r}-00'$$

$$\bar{r}'=\bar{r}-\bar{v}t \qquad (1)$$

Since in NM, time is same in all inertial frame $t'=t$ (2)

The transformation equation given by (1) and (2) are called General Galian Transformation.

If O' is moving alone the common x $x'-axis$ then $\bar{v}=(v,0,0)$

$$(x',y',z')=(x,y,z)-(v,0,0)t$$

$$\Rightarrow x'=x-vt, y'=y,'z'=z,t'=t.$$

These are called simple Galilean Transformation .

Inverse of G.T obtain as

$$x=x'+vt,\ \ y=y',\ \ z=z',\ \ t=t'$$

These are inverse of Galilean Transformation obtain.

Q 3: Show that Newton's Kinamatical Equation are invariant under G.T.

Proof: Consider a particle moving in inertial frame S. It motion is described by Newton's kinematic eqations are

$$V = U + At \quad s = Ut + \frac{1}{2}At^2 \quad V^2 = U^2 + 2As \qquad (1)$$

Where U be the initial velocity particle at time t=0

V be the velocity of particle at time

A be the acceleration of a particle

S be the distance travelled by partical during time 't'

We describe motion of particle from inertial frame S with velocity 'v' along the *x*-axis of S.

Use, $t = t'$, $A = A'$ from (1)

$$V = U + A't' \quad s = Ut' + \frac{1}{2}At'^2 \quad V^2 = U^2 + 2As' \qquad (2)$$

We have $\overline{U'} = \overline{U} - \overline{V}$

Where, U is the velocity of particle measure in S.

Also, We have G.T

$$\Rightarrow x' = x - vt, y' = y, ' z' = z, t' = t.$$

$$\Rightarrow s' = s - vt, y' = y, ' z' = z, t' = t. \qquad (3)$$

From special G.T.

$$\Rightarrow \overline{V'} = \overline{V} - \overline{U} \qquad \overline{U'} = \overline{U} - \overline{V}$$

$$\Rightarrow \overline{V'} - \overline{U'} = \overline{V} - \overline{U}$$

$$\Rightarrow \overline{V'} - \overline{U'} = At$$

$$\Rightarrow \overline{V'} - \overline{U'} = A't'$$

$$\Rightarrow V' = \overline{U'} - A't' \qquad (4)$$

From equation(3), we get $s' = s - Ut,$

$$= Ut + \frac{1}{2}At^2 - Ut$$

$$= (U - V)t + \frac{1}{2}At^2$$

$$S' = (U - V)t + \frac{1}{2}A't'^2 \qquad (5)$$

Equation(4) we get, $(V')^2 = (U' + A't')^2$

$$= U'^2 + 2V'A't' + A'^2t'^2$$

$$= U'^2 + 2A'(U't' + \frac{1}{2}A't'^2)$$

$$(V')^2 = (U'^2 + 2A'S')^2 \qquad (6)$$

Hence equations (4), (5) & (6), observed that are all quantities exactly same in S' frame as the in S frame. Hence, Newton's kinematical equation are invariant under G.T.

Q.4) Prove that $\nabla^2 F - \frac{1}{C^2}\frac{\partial F}{\partial t^2} = 0$ do not remain invariant under G.T.

Solution: we have , $x' = x - vt, \quad y' = y, \quad z' = z, \quad t' = t \qquad \ldots (1)$

Also $x' = x'(x,t) \qquad \ldots (2)$

Consider a function defined by

$$F = F(x', y', z', t') \qquad \ldots (3)$$

From the elementary calculus

$$\frac{\partial F}{\partial x} = \frac{\partial F}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial F}{\partial y'}\frac{\partial y'}{\partial y} + \frac{\partial F}{\partial z'}\frac{\partial z'}{\partial z} + \frac{\partial F}{\partial t'}\frac{\partial t'}{\partial t}$$

$$= \frac{\partial F}{\partial x}(1) + 0 + 0 + 0$$

$$\frac{\partial F}{\partial x} = \frac{\partial F}{\partial x'} \quad \Rightarrow \quad \frac{\partial^2 F}{\partial x^2} = \frac{\partial^2 F}{\partial x'^2} \qquad \ldots (4)$$

Similarly, we can say that,

$$\frac{\partial^2 F}{\partial y^2} = \frac{\partial^2 F}{\partial y'^2} \quad , \quad \frac{\partial^2 F}{\partial z^2} = \frac{\partial^2 F}{\partial z'^2} \qquad \ldots (5)$$

Now , $$\frac{\partial F}{\partial t} = \frac{\partial F}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial F}{\partial y'}\frac{\partial y'}{\partial t} + \frac{\partial F}{\partial z'}\frac{\partial z'}{\partial t} + \frac{\partial F}{\partial t'}\frac{\partial t'}{\partial t}$$

$$= \frac{\partial F}{\partial x'}(-v) + 0 + 0 + \frac{\partial F}{\partial t'}(1)$$

$$\frac{\partial F}{\partial t} = -v\frac{\partial F}{\partial x'} + \frac{\partial F}{\partial t'}$$

Similarly

$$\frac{\partial^2 F}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial F}{\partial t}\right)$$

$$\frac{\partial^2 F}{\partial t^2} = \left(-v\frac{\partial}{\partial x'} + \frac{\partial}{\partial t'}\right)\left(-v\frac{\partial F}{\partial x'} + \frac{\partial F}{\partial t'}\right)$$

$$\frac{\partial^2 F}{\partial t^2} = v^2\frac{\partial^2 F}{\partial x^2} - 2v\frac{\partial^2 F}{\partial x'\partial t'} + \frac{\partial^2 F}{\partial t^2} \qquad \ldots (6)$$

Now consider,

$$\nabla^2 F - \frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} = \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} \qquad \ldots (7)$$

Using (4), (5), (6) in (7) we get,

$$\nabla^2 - \frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} = \frac{\partial^2 F}{\partial x'^2} + \frac{\partial^2 F}{\partial y'^2} + \frac{\partial^2 F}{\partial z'^2} - \frac{1}{c^2}\left(v^2 \frac{\partial^2 F}{\partial x'^2} - 2v\frac{\partial^2 F}{\partial x' \partial t'} + \frac{\partial^2 F}{\partial t^2} \right)$$

$$= \left(\frac{\partial^2 F}{\partial x'^2} + \frac{\partial^2 F}{\partial y'^2} + \frac{\partial^2 F}{\partial z'^2} - \frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} \right) - \frac{1}{c^2}\left(v^2 \frac{\partial^2 F}{\partial x'^2} - 2v\frac{\partial^2 F}{\partial x' \partial t'} \right)$$

$$\Rightarrow \left(\nabla^2 F - \frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} \right) = \left(\nabla'^2 F - \frac{1}{c^2}\frac{\partial^2 F}{\partial t'^2} \right) - \frac{1}{c^2}\left(v^2 \frac{\partial^2 F}{\partial x'^2} - 2v\frac{\partial^2 F}{\partial x' \partial t'} \right)$$

$$\Rightarrow \nabla^2 F - \frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} \neq \nabla'^2 F - \frac{1}{c^2}\frac{\partial^2 F}{\partial t'^2} \ , \quad \text{Unless } v = 0$$

Thus wave equation and Maxwell’s equation do not remains invariant under G.T.

Explain relativity theory

The Law of physics should not change from frame to frame. This forms the fundamental principle of physics & is termed as principle of relativity (P.R).

Maxwell's electromagnetic theory or NM gives the following alternatives:-

1]. The PR applies to N .mechanics & don't electromagnetic theory .Thus

PR become too restrictive & is not universal in physics.

2]. PR apply to N.M & Maxwell's electromagnetic theory but electromagnetic theory of Maxwell needs modification.

3]. The PR is applicable to NM & Maxwell's electromagnetic theory but NM needs revision.

The special relativity (S.R) based on two postulates:-

- **I)** Einstein Principle relativity (E.P.R)
- **II)** Constantacy speed of light

Einstein Principle relativity: According to this roseate all Law of physics are some in all inertial frames. It means that low of physics must remain invariant with respect to the transformation of co-ordinate & time from one inertial frame to other frame.

It is impossible to measure (detect) the accelerated translatory motion of system the free spaces (ether) like medium which must be assume to pervade it.

Principle of constancy of speed of light

According to this principle the speed of light in free space is some for all inertial observers & doesn't depend on relativity velocity of source of light & observer speed of light is denoted by 'C' light propagated in all direction with speed 'C' is given by:

$$x^2 + y^2 + z^2 = c^2t^2$$

(1)

Time dilation :-

Consider a clock C' at rest in S' at appoint on X' axis having coordinate $x'=x'_1$. Assume that when this clock records time $t'=t'_1$.

The standard clock in S which C' is pssing by at the movement record the time t_1 .

At a later time when C ' record the time t_2 it coincides with another clock in S Which record the time t_2

Note that the time t_2 and t_2 are recorded on the same clock C' in S' at the same point while corresponding time t_1 and t_2 are recorded on the same clocks S at two different points . Using SLT , We get

$$t_1 = \alpha(t_1^{'} + \frac{v}{c^2} x_1^{'})$$

$$t_2 = \alpha\left(t_2^{'} + \frac{v}{c^2} x_1^{'}\right)$$

$$t_2 - t_1 = (t_2^{'} - t_1^{'})$$

$$\Delta t = \alpha \Delta t^{'} , \qquad Where\ \Delta t = t_2 - t_1 ,$$

$$\Delta t^{'} < \Delta t .$$

Hence a clock at rest in S' lags behind the clocks in S. As the system S and S' are equivalent we get the symmetrical situation : a clock at rest in S will lag behind the clock in S'. Thus the conclusion is "moving clocks go slow " , This was experimentally verified by Hafele and Keating (1972) .

Simultaneous Events

The events occur at the one and same point and same time are called as simultaneous event. Let two events occur at the points (x_1, y_1, z_1) and (x_2, y_2, z_2) these two events occur in at one and the same point if occur $x_1 = x_2, y_1 = y_2, z_1 = z_2$

$$\Rightarrow x_2 - x_1 = 0, y_2 - y_1 = 0, z_2 - z_1 = 0$$

$$\Rightarrow dx = 0, dy = 0, dz = 0$$

$$\Rightarrow dx^2 + dy^2 + dz^2 = 0$$

$$\Rightarrow dl^2 = 0$$

$$\Rightarrow dl = 0$$

Hence the events occur at same point $\Leftrightarrow dl = 0$

Similarly events occurs at same time $\Leftrightarrow dt = 0$

Theorem 1

Q.9) Show that there exist and inertial system S' in which the two events occurs at once and the same point if the inertial between two events is time like.

Solution:-Let, ds be the time like interval between two events occurring in on inertial system S'. $\Rightarrow ds^2 > 0$

$$\Rightarrow ds^2 = -dx^2 - dy^2 - dz^2 + c^2dt^2 > 0$$

$$= -\left(dx^2 + dy^2 + dz^2\right) + c^2dt^2 > 0$$

$$\Rightarrow ds^2 = -dl^2 + c^2dt^2 > 0$$

For an observer in S' we get

$$ds'^2 = -dl'^2 + c^2dt'^2 \qquad (1)$$

Under Lorentz transformation in ds^2 is invariant

$$\Rightarrow ds^2 = ds'^2 \qquad (2)$$

From (1) and (2) we get

$$ds'^2 = -dl'^2 + c^2dt'^2$$

$$\Rightarrow -dl'^2 + c^2dt'^2 > 0$$

$$\Rightarrow dl'^2 = 0$$

$$\Rightarrow dl' = 0$$

Events happened at once and the same point in S'.

Corollary

Q.10) Show that two events are separated by time like interval can not occur simultaneously in any inertial system.

Proof:-Let ds be the time like interval between two events occurring in an inertial system s.

$$\Rightarrow ds^2 > 0$$

$$ds^2 = dx^2 - dy^2 - dz^2 + c^2 dt^2 > 0$$

$$= -(dx^2 + dy^2 + dz^2) + c^2 dt^2 > 0$$

$$\Rightarrow ds^2 = -dl^2 + c^2 dt^2 > 0$$

For an observer in S' weget

$$ds'^2 = -dl'^2 + c^2 dt'^2$$

$$\Rightarrow -dl'^2 + c^2 dt'^2 > 0$$

$$\Rightarrow dt' \neq 0$$

$\Rightarrow$ Events are not simultaneous in S' but S' is any inertial frame.

$\Rightarrow$ The events cannot be simultaneous in any inertial frame.

Theorem :

Q.12) Show that there exist an inertial system S' which the two events occur at once and the same points if interval between two events is space like.

Solution:- Let *ds* be an space like interval between two events occurring in an inertial system s.

$$\Rightarrow ds^2 > 0$$

$$\Rightarrow ds^2 = -dx^2 - dy^2 - dz^2 + c^2 dt^2 < 0$$

$$= -\left(dx^2 + dy^2 + dz^2\right) + c^2 dt^2 < 0$$

$$\Rightarrow ds^2 = -dl^2 + c^2 dt^2 < 0$$

For an observer in S' we get

$$ds'^2 = -dl'^2 + c^2 dt'^2 \qquad (1)$$

Under Lorentz transformation in ds^2 is invariant

$$\Rightarrow ds^2 = ds'^2 \qquad (2)$$

From (1) and (2) we get

$$ds^2 = -dl'^2 + c^2 dt' < 0$$

$$\Rightarrow dt'^2 = 0$$

$\Rightarrow$ Events occurring at once and the same time in S'.

Q.13) Show that the interval or metric ds^2 between two elements is given by) $ds^2 = -dx^2 - dy^2 - dz^2 + c^2 dt$ **& hence prove that ds^2 is invariant under L.T**.

Solution:-In Euclidian geometry if, $P(x_1, y_1, z_1)$ & Q(x_2, y_2, z_2)then distance between point P&Q is ds=PQ is given by Pythagoras rules.

$$ds^2 = PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \quad (1)$$

If P and Q are neighboring point

i.e. P(x_1, y_1, z_1) and Q$(x + dx, y + dy, z + dz)$

then equation (1) can be written as:

$$ds^2 = (x + dx - x)^2 + (y + dy - y)^2 (z + dz - z)^2$$

$$= (dx)^2 + (dy)^2 + (dz)^2$$

$$= dx^2 + dy^2 + dz^2 \quad (2)$$

Putting the $x = x^1, y = x^2, z = x^3$ equation (2), we get

$$\Rightarrow ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \quad (3)$$

Comparing equation (3) with $ds^2 = g_{mn} dx^m dx^n$

$$g_{11} = g_{22} = g_{33} = 1 \quad (4)$$

Consider inertial an inertial system s in which two events occurs, one event consist of sending light signal at a time t_1 from the space point x_1, y_1, z_1 and second one consisting receiving the same at later time t_2 at point (x_2, y_2, z_2).

If the velocity of light is c.

Then, $c^2(t_2 - t_1)^2 + (y_2 - y_1) + (z_2 - z_1)^2$ (5)

If the events are neighboring

$$x_1 = x \ , \ x_2 = x + dx \ \text{, and } t_1 = t \ , \ t_2 = t + dt$$

Putting equation (5) , we get

$$c^2(t + dt - t)^2 = (x + dx - x)^2 + (y + dy - y)^2 + (z + dz - z)^2$$

$$c^2 dt^2 = dx^2 + dy^2 + dz^2$$

$$\Rightarrow -dx^2 - dy^2 - dz^2 + c^2 dt^2 = 0$$

Defined $ds^2 = -dx^2 - dy^2 - dz^2 + c^2 dt^2$ (6)

Write $x = x^1$, $y = x^2$, $z = x^3$, $ct = x^4$

Putting in equation (6)

$\Rightarrow ds^2 = (dx^1)^2 - (dx^2)^2 - (dx^3)^2 + (dx^4)^2$ (7)

Comparing equation (7) with $ds^2 = g_{mn} dx^m . dx^n$

$g_{11} = g_{22} = g_{33} = 1 \& \ g_{44} = 1$ (8)

The metric equation (7) is the minkowskian metric.

Hence ds^2 is the invariant under Lorentz Transformation.

Q.15:-Show that $x^2+y^2+z^2-c^2t^2$ is Lorentz invariant.

Solution:-

We have special inverse Lorentz transformation,

$$x=\alpha(x'+vt'),y=y',z=z',t=\alpha\left(t'+\frac{v}{c^2}x'\right)$$

To show that, $x^2+y^2+z^2-c^2t^2$ is invariant under special Lorentz transformation

Now,

$$\Rightarrow x^2+y^2+z^2-c^2t^2$$

$$\Rightarrow \alpha^2(x'+vt')^2+y'^2+z'^2-c^2\alpha^2\left(t'+\frac{v}{c^2}x'\right)^2$$

$$\Rightarrow \alpha^2\left(x'^2+2x'vt'+v^2t'^2\right)+y'^2+z'^2-c^2\alpha^2\left(t'^2+2t'x'\frac{v}{c^2}+\frac{v^2}{c^4}x'^2\right)$$

$$\Rightarrow \alpha^2x'^2+2\alpha^2x'vt'+\alpha^2v^2t'^2+y'^2+z'^2-c^2\alpha^2t'^2-2\alpha^2t'x'v-\alpha^2\frac{v^2}{c^2}x'^2$$

$$\Rightarrow \alpha^2x'^2\left(1-\frac{v^2}{c^2}\right)+\alpha^2t'^2\left(v^2-c^2\right)+y'^2+z'^2$$

$$\Rightarrow x'^2\left[\frac{1}{\left(1-\frac{v^2}{c^2}\right)}\left(1-\frac{v^2}{c^2}\right)\right]+y'^2+z'^3-t'^2\frac{c^2}{\left(1-\frac{v^2}{c^2}\right)}\left(1-\frac{v^2}{c^2}\right)$$

$$\Rightarrow x'^2+y'^2+z'^2-c^2t'^2$$

Q 15:-Prove that $\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is invariant under simple Lorentz transformation.

Solution :-We have $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$$\begin{aligned}&\text{And } x = x(x', t')\\ &\Rightarrow x = \alpha(x' + vt')\\ &\quad y = y', z = z'\\ &t = t(x', t')\\ &\Rightarrow t = \alpha\left(t' + \frac{v}{c^2}x'\right)\\ &\Rightarrow t' = \alpha\left(t - \frac{v}{c^2}x\right)\end{aligned}$$

Now,

$$\begin{aligned}\frac{\partial}{\partial x} &= \frac{\partial}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial}{\partial t'}\frac{\partial t'}{\partial x}\\ &= \frac{\partial}{\partial x'}(\alpha) + \frac{\partial}{\partial t'}\left(0 - \alpha\frac{v}{c^2}\right)\\ &= \frac{\partial}{\partial x'}(\alpha) - \frac{\partial}{\partial t'}\frac{\alpha v}{c^2}\\ &= \alpha\left(\frac{\partial}{\partial x'} - \frac{v}{c^2}\frac{\partial}{\partial t'}\right) \qquad (1)\end{aligned}$$

$$\begin{aligned}\frac{\partial^2}{\partial x^2} &= \frac{\partial}{\partial x}\cdot\frac{\partial}{\partial x}\\ &= \alpha\left(\frac{\partial}{\partial x'} - \frac{v}{c^2}\frac{\partial}{\partial t'}\right)\cdot\alpha\left(\frac{\partial}{\partial x'} - \frac{v}{c^2}\frac{\partial}{\partial t'}\right)\\ &= \alpha^2\left(\frac{\partial^2}{\partial x'^2} - 2\frac{v}{c^2}\frac{\partial^2}{\partial x'\partial t'} + \frac{v^2}{c^4}\frac{\partial^2}{\partial t'^2}\right) \qquad (2)\end{aligned}$$

Now,

$$\begin{aligned}\frac{\partial}{\partial y'} &= \frac{\partial}{\partial y'}\cdot\frac{\partial y'}{\partial y}\\ &= \frac{\partial}{\partial y'}(1)\\ &= \frac{\partial}{\partial y'} \qquad (3)\end{aligned}$$

Similarly,

$$\frac{\partial^2}{\partial z'^2} = \frac{\partial}{\partial z'} \cdot \frac{\partial z'}{\partial z}$$

$$= \frac{\partial}{\partial z'} \tag{4}$$

Finally,

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial x'} \cdot \frac{\partial x'}{\partial t} + \frac{\partial}{\partial t'} \cdot \frac{\partial t'}{\partial t}$$

$$= \frac{\partial}{\partial x'}(-\alpha v) + \frac{\partial}{\partial t'}(\alpha)$$

$$= \alpha\left(\frac{\partial}{\partial t'} - \frac{\partial}{\partial x'} v\right)$$

$$\frac{\partial^2}{\partial t^2} = \alpha\left(\frac{\partial}{\partial t'} - \frac{\partial}{\partial x'} v\right) \cdot \alpha\left(\frac{\partial}{\partial t'} - \frac{\partial}{\partial x'} v\right)$$

$$= \alpha^2\left(v^2 \frac{\partial^2}{\partial x'^2} - 2v \frac{\partial^2}{\partial x' \partial t'} + \frac{\partial^2}{\partial t'^2}\right) \tag{5}$$

To show that,

$$\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

$$\Rightarrow \alpha^2\left(\frac{\partial^2}{\partial x'^2} - 2\frac{v}{c^2}\frac{\partial^2}{\partial x' \partial t'} + \frac{v^2}{c^4}\frac{\partial^2}{\partial t'^2}\right) + \frac{\partial}{\partial y'} + \frac{\partial}{\partial z'} - \frac{1}{c^2}\left(v^2 \frac{\partial^2}{\partial x'^2} - 2v \frac{\partial^2}{\partial x' \partial t'} + \frac{\partial^2}{\partial t'^2}\right)$$

$$\Rightarrow \alpha^2 \frac{\partial^2}{\partial x'^2} - 2v\frac{\alpha^2}{c^2}\frac{\alpha^2}{\partial x' \partial t'} + \alpha^2 \frac{v^2}{c^4}\frac{\partial^2}{\partial t'^2} + \frac{\partial}{\partial y'} + \frac{\partial}{\partial z'} - \alpha^2 \frac{v^2}{c^2}\frac{\partial^2}{\partial x'^2} + 2v\frac{\alpha^2}{c^2}\frac{\partial^2}{\partial x' \partial t'} - \frac{\alpha^2}{c^2}\frac{\partial}{\partial}$$

$$\Rightarrow \alpha^2 \frac{\partial^2}{\partial x'^2}\left(1 - \frac{v^2}{c^2}\right) + \frac{\partial}{\partial y'} + \frac{\partial}{\partial z'} - \frac{\alpha^2}{c^2}\frac{\partial^2}{\partial t'^2}\left(1 - \frac{v^2}{c^2}\right)$$

$$\Rightarrow \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \frac{\partial^2}{\partial z'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2}$$

$$\Rightarrow \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2}$$

$\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2}$ is Lorentz transformation.

RELATIVISTIC KINEMATIC'S

Q17):Drive expression for transformation of particle velocity .

Solution:Consider two inertial frames s & s^1 such that s^1 moving uniformly velocity v relative for s along x axis

We have simple Lorentz tansformation

$$x^| = \alpha(x - vt) \quad dy = \alpha dy^| \quad dz = dz^|$$

$$x^| = \alpha(x - vt) \quad y^| = y \quad z^| = z \quad t^| = \alpha(t - \frac{v}{c^2}x) \tag{1}$$

The path of moving particle in s given by

$$x = x(t) \quad y = y(t) \quad z = z(t) \tag{2}$$

The path of moving particle in s^1 given by

$$x' = x(t) \quad y^| = y^|(t) \,\&\, z^| = z^|(t). \tag{3}$$

Let, $u^|$ be the velocity of particle measure in $s^|$ then

$$U^| = (Ux^| Uy^| Uz^|) = \left(\frac{dx^|}{dt^|}, \frac{dy^|}{dt^|}, \frac{dz^|}{dt^|}\right) \tag{4}$$

$$\overline{U}^{|2} = Ux^{|2} + Uy^{|2} + Uz^{|2} \tag{5}$$

Let $\overline{u}^|$ be the velocity of particle measure in s then

$$U^| = (Ux, Uy, Uz) = (\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}) \tag{6}$$

$$\overline{U}^2 = Ux^2 + Uy^2 + Uz^2 \tag{7}$$

To find $Ux^|, Uy^|$ & $Uz^|$

Now,

$$Ux^| = \frac{dx^|}{dt^|}$$

$$Ux^| = \frac{\alpha(dx - vdt)}{\alpha(dt - \frac{v}{c^2}dx)}$$

$$Ux^{|} = \frac{d_x/d_y - v}{1 - \frac{v}{c^2} d_x/d_t}$$

$$Ux^{|} = \frac{u_x - v}{1 - \frac{v}{c^2}.u_x} \qquad (8)$$

$$Uy^{|} = \frac{dy^{|}}{dt^{|}}$$

$$Uy^{|} = \frac{d_y}{\alpha\left(dt\left(v/c^2\, d_x\right)\right)}$$

$$Uy^{|} = \frac{d_y/d_t}{\alpha\left(1 - v/c^2\, u_x\right)} \qquad (9)$$

Simillary we obtain equation 10) the transformation of partical velocity

$$Uz^{|} = \frac{d_y/d_z}{\alpha\left(1 - v/c^2\, u_x\right)} \qquad (10)$$

Note that invers transformation of this velocity are given by interchanging the prime and unprimed quantity replacing v=-v/c

$$Ux = \frac{u^{|}{}_x + v}{1 + \frac{v}{c^2}.u_x} \qquad Uy = \frac{u_y^{|}}{\alpha\left(1 + \frac{v}{c^2} u_x^{|}\right)} \qquad Uz = \frac{u_z^{|}}{\alpha\left(1 + \frac{v}{c^2} u_x^{|}\right)}$$

Q18) : Derive expression for transformation of particle.

Proof: Consider two inertial frames s and s' moving uniformly with velocity v relative to s along their common xx'-axis.
We have SLT,

$$x' = \alpha(x - vt) \;\; y' = y\,, \;\; z' = z\,, \;\; t' = \alpha\left(t - \frac{v}{c^2}\right) \qquad (1)$$

The path of moving particle in s is given by,

$$\text{Then} \quad x = x(t), \;\; y = y(t), \;\; z = z(t) \qquad (2)$$

And the path in s' is given by ,

$$x' = x'(t), \;\; y' = y'(t), \;\; z' = z'(t) \qquad (3)$$

Let, $\overline{u'}$ be the velocity of particle measure in s', then

$$\overline{u'} = (u'_x, u'_y, u'_z)$$

$$\overline{u'} = \left(\frac{dx'}{dt'}, \frac{dy'}{dt'}, \frac{dz'}{dt'}\right) \qquad (4)$$

$$\overline{u'}^2 = \left(u'^2_x + u'^2_y + u'^2_z\right) \qquad (5)$$

Let $\bar{u}$ be the velocity of particle measured in s, then

$$\bar{u} = (u_x, u_y, u_z)$$

$$\bar{u} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) \qquad (6)$$

$$\bar{u}^2 = \left(u_x^2 + u_y^2 + u_z^2\right) \qquad (7)$$

To find , u'_x, u'_y, u'_z

Now,

$$u'_x = \frac{dx'}{dt'}$$

$$u'_x = \frac{\alpha(dx - v\,dt)}{\alpha\left(dt - \frac{v}{c^2}dx\right)}$$

$$u'_x = \frac{\left(\frac{dx}{dt} - v\right)}{\left(1 - \frac{v}{c^2}\frac{dx}{dt}\right)}$$

$$u'_x = \frac{(u_x - v)}{\left(1 - \frac{v}{c^2}u_x\right)} \qquad (8)$$

$$u'_y = \frac{dy'}{dt'}$$

$$u'_y = \frac{dy}{\alpha\left(dt - \frac{v}{c^2}dx\right)}$$

$$u'_z = \frac{u_z}{\alpha\left(1 - \frac{v}{c^2}u_x\right)}$$

$$u'_x = \frac{\left(\frac{dy}{dt}\right)}{\alpha\left(1 - \frac{v}{c^2}\frac{dx}{dt}\right)}$$

$$u'_y = \frac{u_y}{\alpha\left(1 - \frac{v}{c^2}u_x\right)} \qquad (9)$$

Similarly, we obtained

$$u'_z = \frac{u_z}{\alpha\left(1 - \frac{v}{c^2}u_x\right)} \qquad (10)$$

The equations (8), (9) and (10) gives the transformation of particle velocity.

Proper time

Q.19).Define proper time and show that moving clocks go slow than those at rest.

Definition: The time recorded by a clock moving with a body is the proper time for the body.

Suppose that a man travelling in a train measures the total travel time for his journey as 3 hours on his clock. This 3 hours time is the proper time for him. Another man on the platform shall record different time for the journey.The proper time can be considered as the time interval between two events happening at the same place or the time interval measured by a single clock at one place. Consequently there is a non-proper or improper time which is a time interval measured by two different clocks at two different points. Let a clock at rest in S' record time dt' between two events. Since the clock is at rest,

$$dx' = dy' = dz' = 0$$

And then we get $ds'^2 = c^2 dt'^2$. (1)

Correspondingly in S frame a clock records time dt and

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 .$$

$$c^2 dt'^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2, \qquad \because ds^2 = ds'^2$$

$$= c^2 dt^2 \left(1 - \frac{1}{c^2} \frac{dx^2 + dy^2 + dz^2}{dt^2}\right)$$

$$= c^2 dt^2 \left(1 - \frac{v^2}{c^2}\right), \qquad \because v^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2,$$

$$dt' = dt\left(1 - \frac{v^2}{c^2}\right)^{1/2}. \qquad (2)$$

Consider two events A and B which are on the world line of a moving clock. Like distance, the time interval between the events depends on the route taken by the clock. Let A and B be joined by two paths:

Path1: consisting of a world line of a clock at rest

Path2: consisting of a closed world line starting from A and returning to it.

Along this path1, 2 gives, after integration, that

$$\int_A^B dt' = \int_A^B \left(1 - \frac{v^2}{c^2}\right)^{1/2} dt$$

$$t'_B - t'_A = \int_A^B dt, \qquad \because v = 0 \text{ For the clock at rest}$$

$$T'_1 = t_B - t_A, \qquad \text{where } T'_1 = t'_B - t'_A. \qquad (3)$$

Hence along the path 1, proper time T'_1 coincides with the time $t_B - t_A$ measured by the rest clock. Similarly along the path2, we write from (2) that

$$T'_2 = \int_A^B \left(1 - \frac{v^2}{c^2}\right)^{1/2} dt = \int_A^B \left(1 - \frac{v^2}{2c^2} + \cdots\right) dt, \qquad \because v << \text{c}$$

$$= \int_A^B dt - \frac{1}{2c^2}\int_A^B v^2 dt$$

$$= (t_B - t_A) - \text{Positive quantity}, \qquad \because v^2 > 0$$

$$= T'_1 - \text{Positive quantity}, \quad \text{by (3)}$$

$$T'_2 < T'_1.$$

Hence the proper time of a moving object is always less than the corresponding interval in the rest system. This means that moving clocks go slow than those at rest.

THE MASS OF A MOVING PARTICLE

Q.21) Derive the expression for *relativistic mass* of the particle

We suppose that the laws of conservation of mass and momentum are valid in two inertial frames S and $S^{'}$, where $S^{'}$ is moving along their common $x-x^{'}$axis with velocity v and the axes $y^{'}$ and $z^{'}$are parallel to $y$and $z$axes respectively. Consider that two exactly identical elastic particles are moving with velocities $+u^{'}$ and $-u^{'}$ relative to $S^{'}$ parallel to $x^{'}$ axis such that a head on collision can takes place. Since, the particles are perfectly elastic, they will come to rest momentarily in $S^{'}$and then rebound under the elastic forces developed such that they move back along their initial paths with velocities $-u^{'}$ and $+u^{'}$in $S^{'}$.

Let u_1 and u_2 be the velocities of these particles before collision.

Applying velocity transformation law, we get

$$u_x = \frac{u_x^{'} + v}{1 + u_x^{'}\dfrac{v}{c^2}}$$

We have:

$$u_1 = \frac{u^{'} + v}{1 + u^{'}\dfrac{v}{c^2}} \;, \qquad \because u_x^{'} = u^{'}, u_x = u_1 \qquad (1)$$

And $$u_2 = \frac{-u^{'} + v}{1 - u^{'}\dfrac{v}{c^2}} \;, \qquad \because u_x^{'} = -u^{'}, u_x = u_2 \qquad (2)$$

The velocities u_1 and u_2 have the same direction in *S*.

Suppose that m_1 and m_2 are the masses of particles before collision in *S*. Let *M* be the sum of masses of the two particles measured in *S* during collision when they come to rest relative to each other. At this instant both the particles are at rest in $S^{'}$ and hence both are moving with velocity *v*relative to *S*.

The conservation of mass gives:-

Total mass before collision = total mass during collision.

$$\Rightarrow m_1 + m_2 = M \qquad (3)$$

Also, onservation of momentum gives:-

Total momentum before collision = Total momentum during collision.

$$\Rightarrow m_1 u_1 + m_2 u_2 = Mv \qquad (4)$$

$$\Rightarrow m_1 u_1 + m_2 u_2 = (m_1 + m_2)v \quad \text{, by (3)}$$

$$m_1(u_1 - v) = m_2(v - u_2)$$

$$\Rightarrow m_1 \left\{ \frac{u' + v}{1 + u' \frac{v}{c^2}} - v \right\} = m_2 \left\{ v - \frac{-u' + v}{1 - u' \frac{v}{c^2}} \right\} \quad \text{, by (1) and (2)}$$

$$\Rightarrow \frac{m_1}{1 + u' \frac{v}{c^2}} u' \left(1 - \frac{v^2}{c^2}\right) = \frac{m_2}{1 - u' \frac{v}{c^2}} u' \left(1 - \frac{v^2}{c^2}\right)$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{1 + u' \frac{v}{c^2}}{1 - u' \frac{v}{c^2}} \qquad (5)$$

Now, by using inverse of Lorentz Contraction Factor:-

We have:-

$$\left(1 + u_x' \frac{v}{c^2}\right) = \left\{ \frac{\left(1 - \frac{v^2}{c^2}\right)\left(1 - \frac{u'^2}{c^2}\right)}{\left(1 - \frac{u^2}{c^2}\right)} \right\}^{\frac{1}{2}} \qquad (6)$$

For particle 1, in S' and S :

$$\text{From (6)} \Rightarrow \left(1 + u' \frac{v}{c^2}\right) = \left\{ \frac{\left(1 - \frac{v^2}{c^2}\right)\left(1 - \frac{u'^2}{c^2}\right)}{\left(1 - \frac{u_1^2}{c^2}\right)} \right\}^{\frac{1}{2}} \qquad (7)$$

For particle 2 , in S' and S : $\quad u_x' = -u', u' = u', u = u_2$

$$(6) \quad \Rightarrow \left(1 - u' \frac{v}{c^2}\right) = \left\{ \frac{\left(1 - \frac{v^2}{c^2}\right)\left(1 - \frac{u'^2}{c^2}\right)}{\left(1 - \frac{u_2^2}{c^2}\right)} \right\}^{\frac{1}{2}} \qquad (8)$$

$$\Rightarrow \frac{\left(1 + u' \frac{v}{c^2}\right)}{\left(1 - u' \frac{v}{c^2}\right)} = \frac{\sqrt{\left(1 - \frac{u_2^2}{c^2}\right)}}{\sqrt{\left(1 - \frac{u_1^2}{c^2}\right)}}$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{\sqrt{\left(1 - \frac{u_2^2}{c^2}\right)}}{\sqrt{\left(1 - \frac{u_1^2}{c^2}\right)}} \quad \text{, by (5)}$$

$$\Rightarrow m_1 \sqrt{\left(1 - \frac{u_1^2}{c^2}\right)} = m_2 \sqrt{\left(1 - \frac{u_2^2}{c^2}\right)} \qquad (9)$$

Now, assume that $m_1 = m_0$ when $u_1 = 0$ and $m_2 = m_0$ when $u_2 = 0$. Then ,

From(9) $\Rightarrow m_1 \sqrt{\left(1 - \frac{u_1^2}{c^2}\right)} = m_2 \sqrt{\left(1 - \frac{u_2^2}{c^2}\right)} = m_0$

$$\Rightarrow m_1 = \frac{m_0}{\sqrt{\left(1 - \frac{u_1^2}{c^2}\right)}} \text{ and } m_2 = \frac{m_0}{\sqrt{\left(1 - \frac{u_2^2}{c^2}\right)}}$$

Hence, to conserve both mass and momentum in *S* during collision , the mass of the particle has to be redefined such that when it moves with velocity *u* relative to *S*, it’s mass is:-

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{u^2}{c^2}\right)}} \qquad (10)$$

Here, m_0 is the mass of the particle when it is at rest. It is called the *rest mass* or *proper mass* of the particle. The quantity *m* in (10) is the *relativistic mass* of the particle. The mass referred to is the *inertial mass* of the particle.

.22) Define force and obtain the expression for the longitudinal mass of the article.

Force : In theory of relativity force F acting on a particle is define as the rate of change of momentum i.e

$$F = \frac{dp}{dt} \qquad (1)$$

$$\Rightarrow F = \frac{d}{dt}(mu) = m\frac{du}{dt} + u\frac{dm}{dt} \qquad (2)$$

the first term on RHS of the above equation is mass*acceleration. The second term is relativistic correction due to dependence of mass on velocity. Since, $m = m(u)$,we have

$$\frac{dm}{dt} = \frac{dm}{dt}\frac{du}{dt} = \frac{du}{dt}\frac{d}{du}\left(\frac{m_0}{\sqrt{1-u^2/c^2}}\right)$$

$$= \frac{du}{dt}\left(\frac{m_0\, u/c^2}{\left(1-u^2/c^2\right)^{\frac{3}{2}}}\right)$$

$$= \frac{m_0/c^2}{\left(1-u^2/c^2\right)^{\frac{3}{2}}}\bar{a}\circ\bar{u}, \because \frac{du}{dt} = |\bar{a}| \qquad (3)$$

$$\text{then} \Rightarrow \bar{F} = \frac{m_0}{\left(1-u^2/c^2\right)^{\frac{1}{2}}}\bar{a} + \frac{(m_0/c^2)\bar{a}\circ\bar{u}}{\left(1-u^2/c^2\right)^{\frac{3}{2}}}\bar{u} \qquad (4)$$

above equation has newtonian analogue $\bar{F} =$ mass*acceleration in the two cases: 1) $\bar{a}$ is perpendicular to $\bar{u}$ and 2) $\bar{a}$ is parallel to $\bar{u}$.

case 1): here $\bar{a} \circ \bar{u} = 0$ then (4) gives

$$\bar{F} = \frac{m_0}{\left(1 - u^2/c^2\right)^{\frac{1}{2}}}\bar{a}$$

Thus $\bar{F}$ = mass*acceleration, where mass is $\frac{m_0}{\left(1 - u^2/c^2\right)^{\frac{1}{2}}}$ .It is called as transverse mass of a particle.

case 2): In this case $\bar{a} = k\bar{u}$, k = constant. Then (4) becomes

$$\bar{F} = \frac{m_0}{\left(1 - u^2/c^2\right)^{\frac{1}{2}}}k\bar{u} + \frac{(m_0/c^2)k\bar{u} \circ \bar{u}}{\left(1 - u^2/c^2\right)^{\frac{3}{2}}}\bar{u}$$

$$= \frac{m_0 k\bar{u}}{\left(1 - u^2/c^2\right)^{\frac{1}{2}}} + \frac{(m_0/c^2)u^2 k\bar{u}}{\left(1 - u^2/c^2\right)^{\frac{3}{2}}}$$

$$= \frac{m_0 k\bar{u}}{\left(1 - u^2/c^2\right)^{\frac{3}{2}}}\left\{1 - \frac{u^2}{c^2} + \frac{u^2}{c^2}\right\}$$

$$= \frac{m_0 k\bar{u}}{\left(1 - u^2/c^2\right)^{\frac{3}{2}}}\bar{a}$$

This is again in the form $\bar{F}$ = mass*acceleration, where mass is $\frac{m_0}{\left(1 - u^2/c^2\right)^{\frac{3}{2}}}$ this mass is called the longitudinal mass of the particle.

Q.23) Obtain the mass energy equivalence expression/ relation.

Proof : In newton mechanics, the K.E of body is total work done by the force acting on the body.

We know that, the work done dw by force $\overline{F}$ in moving body through a displacement $d\overline{r}$ is

$$dw = \overline{F} \cdot d\overline{r}$$

Assume that all these work goes into increase of K.E T of the particle.

$$dT = \overline{F} \cdot d\overline{r}$$

$$\frac{dT}{dt} = \overline{F} \cdot \frac{d\overline{r}}{dt}$$

$$\frac{dT}{dt} = \overline{F} \cdot \overline{u} \qquad (1)$$

Where u is the velocity of the particle.

$$\frac{dT}{dt} = \left(m\frac{d\overline{u}}{dt} + \overline{u}\frac{dm}{dt} \right).u$$

$$\frac{dT}{dt} = m\frac{d\overline{u}}{dt} \cdot \overline{u} + \overline{u}\,\overline{u}\frac{dm}{dt}$$

$$\frac{dT}{dt} = m\overline{u}\frac{du}{dt} - u^2\frac{du}{dt}\frac{m_0 u/c^2}{\left(1-u^2/c^2\right)^{3/2}}$$

$$\frac{dT}{dt} = \frac{m_0}{\left(1-u^2/c^2\right)^{1/2}}\overline{u}\frac{du}{dt} + u^3\frac{du}{dt}\frac{m_0 u/c^2}{\left(1-u^2/c^2\right)^{3/2}}$$

$$\frac{dT}{dt} = \frac{m_0 u}{\left(1-u^2/c^2\right)^{3/2}}\frac{du}{dt}$$

$$\frac{dT}{dt} = \frac{d}{dt}\frac{m_0c^2}{\left(1-u^2/c^2\right)^{1/2}}$$

Integrating on both side from t=0 and t=T and
Taking u=0 and u=u

$$\frac{dT}{dt} = \frac{m_0c^2}{\left(1-u^2/c^2\right)^{1/2}} - m_0c^2 \quad (2)$$

$$T = \left(m - m_0\right)c^2$$

$$T = mc^2 - m_0c^2 \quad (3)$$

We expand R.H.S of equation (2) in the power of $\left(\frac{u}{c}\right)^2$

$$T = m_0c^2\left(1 + \frac{1}{2}\frac{u^2}{c^2} + \frac{3}{8}\frac{u^4}{c^4}\cdots\right) - m_0c^2$$

$$T \cong \frac{1}{2}m_0c^2 \quad , \forall u << c$$

We define,

$$E = m_0c^2 + T$$

$$T = E - m_0c^2 \quad (4)$$

Substituting equation (4) in (3), we get

$$E - m_0c^2 = mc^2 - m_0c^2$$

$$\mathrm{E} = mc^2$$

This formula is known as Einstein formula showing that two fundamental conception of mass
and energy are identified.

Q.24) Define Four velocity and four acceleration and Hence u^i is a unit vector with magnitude positive and thus it is timelike

The four velocity of a partical is defined as

$$u^i = \frac{dx^i}{ds}.$$

According the four acceleration is

$$a^i = \frac{du^i}{ds}$$

Theorem 1. Prove that the four velocity of a partical is a unit timelike vector.

Proof. $ds^2 = g_{ij}dx^i dx^j$

$$\Rightarrow 1 = g_{ij}\frac{dx^i}{ds}\frac{dx^j}{ds}$$

$$\Rightarrow 1 = g_{ij}u^i u^j$$

Hence u^i is a unit vector with magnitude positive and thus it is timelike.

Q 25): Show that the four velocity and four acceleration are mutually orthogonal.

Solution: Since u^i is a unit vector, by definition,

we have, $g_{ij}u^i u^j = 1$

$$\Rightarrow \frac{d}{ds}\left(g_{ij}u^i u^j\right) = 0$$

$$\Rightarrow g_{ij}\left(\frac{du^i}{ds}u^j + u^i\frac{du^j}{ds}\right) = 0 \because g_{ij} \text{ are constants}$$

$$\Rightarrow g_{ij}a^i u^j + g_{ij}u^i a^j = 0$$

$$\Rightarrow g_{ij}a^i u^j + g_{ij}u^i a^j = 0 \text{ where } i \leftrightarrow j \text{ in second trem}$$

$$\Rightarrow 2g_{ij}a^i u^j = 0, \because g_{ij} = g_{ji}$$

$$\Rightarrow g_{ij}a^i u^j = 0$$

Hence a^i and u^j are orthogonal vectors.

Thank you.